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مادة [5]

### \* Stochastic (Random) Variables :-

Consider the experiment of rolling a pair of fair dice. Let  $X$  denotes the sum rolled. Evidently  $X$  is a Stochastic variable which takes the values

$$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

each with certain probability. For example

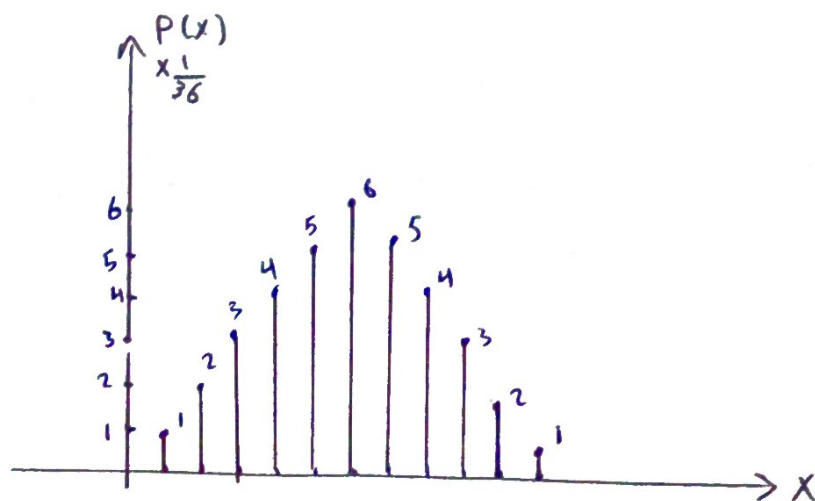
Let  $E_1$ : The sum being 6.

$$\text{Then } E_1 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$$

$$P(E_1) = \left\{ \left( \frac{1}{6} \cdot \frac{1}{6} \right) + \dots \right\} = \frac{5}{36}$$

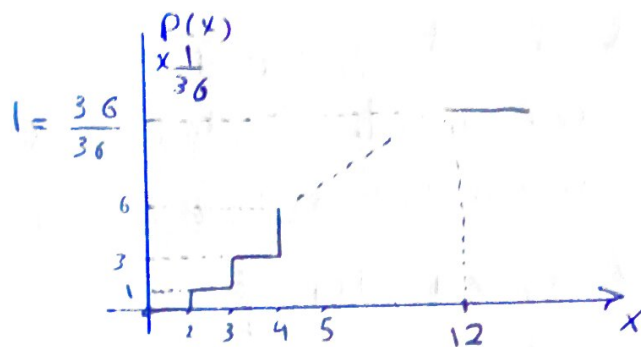
The probabilities of other sums are shown in the table and figure

$X$	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$



## \* Cumulative Distribution function

Based on the previous figure:-



It is defined as a function  $F(x)$  equals the probability that the stochastic variable  $X$  takes values less than or equal to a value  $x$ :

$$F(x) = P(X \leq x)$$

For example  $F(5) = P(X \leq 5)$

The distribution function is an increasing function and has the following properties:-

i -  $F(-\infty) = 0$

ii -  $F(\infty) = 1$

iii -  $F(a) \leq F(b)$  for  $a \leq b$

If  $X$  is a discrete random variable with probability  $P$ , then  $F(x)$  is the step function defined by

$$F(x) = \sum_{x_i \leq x} P(x_i)$$

$\Rightarrow$  Turn over

on the other hand, if  $X$  is a continuous random variable with distribution  $P$ , then

$$F(x) = \int_{-\infty}^x p(t) dt$$

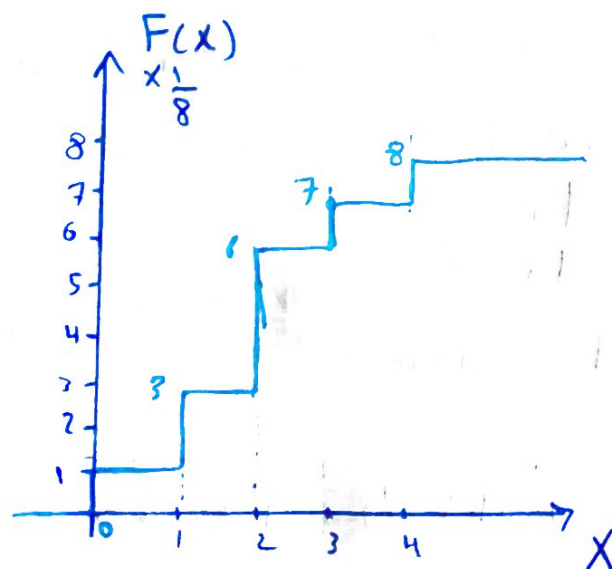
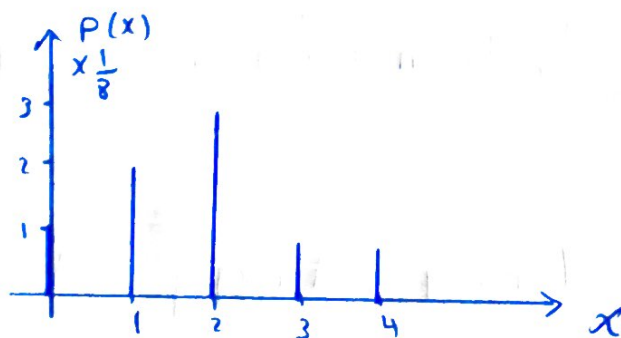
In either cases,  $F(x)$  is monotonic increasing

Example:- Let  $x$  be a discrete random variable with the probability function

$x$	0	1	2	3	4
$P(x)$	$1/8$	$2/8$	$3/8$	$1/8$	$1/8$

$P(x) = 0$  elsewhere, find the comm. dist. fn.

$$F(-1) = F(x \leq -1) = F(-\infty) = 0$$





$$F(-1) = F(X \leq -1) = F(-\infty) = 0$$

$$F(0) = P(X \leq 0) = \frac{1}{8}, F(1/2) = P(X \leq 1/2) = \frac{1}{8}$$

$$F(1) = P(X \leq 1) = P(0) + P(1) = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

$$F(2) = P(X \leq 2) = P(0) + P(1) + P(2) = \dots = \frac{6}{8}$$

$$F(3) = P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = \frac{7}{8}$$

$$F(4) = P(X \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$F(5) = F(6) = \dots = F(\infty) = 1$$

$$\text{CK: } F(-\infty) = 0, F(\infty) = 1, F(1) < F(2), F(1.5) = F(1)$$

\* Density Function: (probability Density  $f_n$ )

(Prob.  $f_n$ )

(Density  $f_n$ )

If  $F(x)$  is a continuous function and if

$f(x) = \frac{dF(x)}{dx}$ , then  $f(x)$  is called the density

function.

$$\int_{-\infty}^x f(x) dx = \int_{-\infty}^x dF(x) = F(x) \Big|_{-\infty}^x = F(x) - F(-\infty) = F(x)$$

- Properties of the density function:-

i -  $f(x) \geq 0$ , hence  $F(x)$  is an increasing function,

$$\text{then } \frac{dF}{dx} \geq 0$$

ii -  $\int_{-\infty}^{\infty} F(x) dx = 1$ , hence

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{\infty} dF(x) = F(x) \Big|_{-\infty}^{\infty} = F(\infty) - F(-\infty) = 1$$

The area under the density curve always equals one

Note: If  $x$  is discrete, then

$$\sum_{-\infty}^{\infty} f(x) = 1$$

Example:-

$$\text{Let } f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- Is  $f(x)$  a density function?
- If so, find the distribution fn  $F(x)$

